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A STOCHASTIC DIFFERENTIAL EQUATION MODEL OF VARIATION OF RICE PRODUCTION

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Abstract: The study assessed the Stochastic Differential Equation (SDE) model of the variation in rice production in Benue State, Nigeria. We considered variation in rice output to be subject to random influence driven by the Wiener process which is a derivative of Brownian motion. The study used secondary data from Benue State Agricultural and Rural Development Authority (BENARDA) for the period of 23 years (1986-2009). The data was analyzed using descriptive statistics to determine the drift and the volatility coefficients which constitutes the basic parameter of an SDE. The resulting model was solved numerically using the Stochastic Runge-Kutta Scheme. Result obtained from our model shows the trend in Rice production and a 5th degree interpolating polynomial was passed through the trajectory of the plot of the simulated data so as to make predictions. The results generated from the simulation were subjected to test for significance using the student T-test for independent data set and was found not to have significant difference. Result of our model agrees with the work of Henri (2007) and Rajoti (2014). However in our work we used parameters from the study area rather than the assumed values used in other work. We conclude that the model is suitable for prediction of rice output.
Keywords: Stochastic differential equation, rice production, wiener process, prediction

Introduction

Stochastic differential equations (SDEs) are differential equations in which one or more of the terms are a stochastic process. The theory of SDEs has been a subject of study for over sixty years and advance in research have made it possible for researcher to model and understand finite dimensional stochastic dynamics using stochastic calculus and the theory of sub-Martingales. Today, stochastic differential equation has found applications in divers fields of human endeavor such as finance, civil and mechanical engineering, economics and environmental sciences, chemistry, physics, signal processing, filtering problems, population dynamics, psychology, medicine and many others (Philip, 2004).

Stochastic calculus is a branch of mathematics that operates on stochastic processes. It allows a consistent theory of integration to be defined for integrals of stochastic processes with respect to stochastic processes. It is used to model systems that behave randomly the best known of which is the Wiener process (named in honour of Norbert Wiener), which is used for modeling Brownian motion as described by Albert Einstein and other physical diffusion processes in space of particles subject to random forces. Since the 1970s, the Wiener process has been widely applied to model a wide range of dynamical systems that are subject to random influence (Allen, 2007).

Mathematical model resulting in SDE can be treated as an Ito or Stratonovich calculus. The Itō calculus is named after Kiyoshi Itō and it extends the methods of calculus to stochastic processes such as Wiener process. The central concept is the Itō stochastic integral which is a generalization of the ordinary concept of a Riemann–Stieltjes integral. The generalization is in two respects. Firstly, if deals with random variables (more precisely, stochastic processes). Secondly, the method allows integration with respect a non-differentiable function (Platen and Bruti-Liberati, 2010). An alternative method to the Ito calculus is the Stratonovich calculus which was introduced by Ruslan L. Stratonovich and D. L. Fisk is the preferred method for modelling stochastic processes in applied mathematics, Klebaner (2005),

Typically, SDEs contain a variable which represents random white noise calculated as a derivative of Brownian motion or the Wiener process. This made SDEs most suited for modeling dynamics system that is subject to random influence. The role of SDEs in modeling continuous time stochastic dynamics can be compared to the role of deterministic ordinary differential equations (ODEs) in non-random differentiable dynamics. Stochastic differential equation is most suited for modeling dynamical systems that is subject to random influence. In Agriculture, the effect of climate change and weather on crops also leads to variability of product which is stochastic.

Changes in climate affect many crops grown around the world. Crops such as wheat and rice grow well in high temperatures, while plants such as maize and sugarcane prefer cooler climates. Changes in rainfall patterns will also affect how well plants and crops grow. The effect of a change in the weather on plant growth may lead to some countries not having enough food. Brazil, parts of Africa, south-east Asia and China will be affected the most and many people could be affected by hunger (Piere Van der, 2004).

The study of the variability of rice output in Benue state entails primarily the formulation of SDE Model to help understand the variability of rice output and predict where necessary the feature rice output in the state. Benue state lies within the lower river Benue trough in the middle belt region of Nigeria with great potentials for rice production. The unique environmental conditions for the state allows for upland and low land production.

In Nigeria, the problem of rice production has been of serious concern to the Government and it citizen at large. With rice now being a structural component of the Nigeria diet and rice import making up an important share of Nigeria agricultural imports, there is need for considerably research interest in increasing local rice production. Studies have shown that year to year variation in crop yields is normally associated with fluctuation in weather. (Ossai, 2004)

Insufficient and high variability in rice yield is a threat to food security in Nigeria. Research carried out on the variability of rice yield in most cases is focused on agricultural, economical point, with general assumptions of coefficient (Yang, 2016). However, it is the assumption of Allen (2007) that a model derived from their practical observation of the behavior of its basic parameters will give a clearer understanding.

Methods and Fomulations

Assumptions and formulation of model

In order to formulate a stochastic model of the variation of rice output in Benue state, we generally assume that variation in rice output is subject to myriad uncertain factors in the environment which we cumulatively model as a Wienaer process, other specific assumptions follows

(i) Variation in rice output as a normal random variable with μ = 0 and σ = 1.

(ii) All farm inputs for rice output are constant.

(iii) Rice production cannot be negative, once a rice production reaches 0 it cannot go any lower.

(iv) The rice production is an adapted process, adapted to rice yield as it evolves overtime.

The parameters used in the model are listed in Table 1.

Table 1: Parameters of the model

Variables/ parameters	Meaning
T	Time of production
R(t)	Rice output at time t.
B(t)	Standard Wiener process
μ	The constant trend or mean output
W_t	The white noise
σ	The constant of volatility or the variance of output
$R(s)$	The rice output in a given interval
$X(t)$	Is an Ito's process
$Z(t)$	Is a function of an Ito's process
f	Is function of a process
R_0	Rice output at initial time
$\alpha(t)$	Is the relative rate of output change at time t

Model equation for the variability of rice output

Following the assumptions, the model is derived using the elementary ordinary differential equation approach

$$\frac{dR(t)}{dt} = \alpha(t)R(t),\ R(0) = R_0 \text{ (constant)} \quad (1)$$

where R(t) is the rice output at time t, and α(t) is the relative rate of output change at time t; we take it that α(t) is not completely known but subject to some random environmental effects so that

$$\alpha(t) = \mu + \text{"}noise\text{"} \quad (2)$$

The behavior of the noise term is not known, only its probability distribution. The function μ is assumed to be nonrandom.

By stochastic process we take,

Noise $= \sigma W_t$ where $W_t =$ is the white noise or Wiener process and $\sigma =$ volatility constant

Therefore equation (2.2) becomes;

$$a(t) = \mu + \sigma W_t \quad (3)$$

Substituting equation (2.3) into equation (2.1) our model become,

$$\frac{dR(t)}{dt} = \mu R(t) + \sigma . W_t R(t) \quad (4)$$

From the equation (2.4) the rice production is without bounds (exponential production), to get a model which conform to reality we introduce the logistic function to get,

$$\frac{dR(t)}{dt} = \mu R(t)(\phi - \frac{R(t)}{K}) + \sigma . W_t R(t) \quad (5)$$

$$\frac{dR(t)}{dt} = \mu R(t)(\phi - \eta R(t) + \sigma . W_t R(t) \quad (6)$$

Where: $K =$ is the maximum target amount of rice and $\eta = \frac{1}{K}$

Where: $R(t)$ is the rice output at time t, where $\mu \in \Re$ and σ >0. This equation (2.6) can be rewritten in integral form as;

$$R_t = R_0 + \mu \int_0^t R(s)(1 - \eta R(s))ds + \sigma \int_0^t R(s)dB_s \quad t \in \Re_+ \quad (7)$$

Determination of drift and volatility

The drift (mean) and volatility (variance) coefficients which are the basic parameters of an SDE model were determined statistically from the simple descriptive statistics ran with the Statistical package for social science (SPSS) package version 21. These were fitted into the model which resulted in the SDE represented in integral form in equation (7).

Numerical solution

Most mathematical models resulting in SDE are rather solved numerically than analytically. This is because only a few SDEs of practical application are explicitly solvable. There is therefore the need for systematic development of efficient numerical methods implementable on digital computers. This according to Peterson (1995) requires simulation of large numbers of differential sample paths so that various statistical properties of the solution could be estimated. Consequently, numerical approach was used as the method of solution to the model formulated.

Here, we shall show the explicit simplification of the stochastic Runge-Kutta scheme in order to solve the formulated stochastic differential equations in equation (6).

Consider the SDE;

$$dZ(t) = f(Z(t),t)dt + g(Z(t),t)dB(t) \quad (8)$$

If

$$K_0 = f(Z_i, t_i),\ G_0 = g(Z_i, t_i),\ i = 0,1,2,\cdots,n-1$$

$$Z_i^{(0)} = Z_i + \frac{1}{2}K_0 \Delta t_i + \frac{1}{2}G_0 \Delta W_i$$

$$K_1 = f(Z_i^{(0)}, t_i + \frac{1}{2}\Delta t_i),\ G_1 = g(Z_i^{(0)}, t_i + \frac{1}{2}\Delta t_i)$$

$$Z_i^{(1)} = Z_i + \frac{1}{2}K_1 \Delta t_i + \frac{1}{2}G_1 \Delta W_i\ K_2 = f(Z_i^{(1)}, t_i + \frac{1}{2}\Delta t_i),$$

$$G_2 = g(Z_i^{(1)}, t_i + \frac{1}{2}\Delta t_i)\ Z_i^{(2)} = Z_i + \frac{1}{2}K_2 \Delta t_i + \frac{1}{2}G_2 \Delta B_i$$

$$K_3 = f(Z_i^{(2)}, t_i + \frac{1}{2}\Delta t_i)\ G_3 = g(Z_i^{(2)}, t_i + \frac{1}{2}\Delta t_i)$$

then the stochastic Runge-Kutta scheme in one dimension is given by;

$$Z_{i+1} = Z_i + \frac{1}{6}(K_0 + 2K_1 + 2K_2 + K_3)\Delta t_i + \frac{1}{6}(G_0 + 2G_1 + 2G_2 + G_3)\Delta W_i \quad (9)$$

For i=0,1,2,$\cdots$,n-1, $Where\ \Delta t_i = t_{i+2} - t_{i+1}$,

$$\Delta W_i = W_{i+2} - W_{i+1}, t_i = i, t_{i+1} = i + 1.$$

Results and Discussion

The output from the descriptive statistics of the data collected is shown in Table 2.

Table 2: Descriptive statistics of the data collected

Variables	N	Minimum	Maximum	Mean	Std. Deviation	Variance
RICE OUTPUT (IN TONNES)	24	171.20	341.88	254.0613	42.90761	1841.063
MEANOUTPUT (IN TONNES)	24	1.07	2.50	2.0467	.31529	.099
AREA	24	81.79	166.70	125.5638	24.68768	609.482
Valid N (listwise)	24					

From the Table, the drift coefficient for the model is 2.0467 and the volatility is 0 .099.
Now, the model in a differential form is:

$$\frac{dR(t)}{dt} = \mu R(t)(\phi - \frac{R(t)}{K}) + \sigma . W_t R(t)$$

Where $\mu R(t)$ = 2.0467 and $\sigma .(R(t))$ = 0 .099. Hence the model equation becomes:

$$\frac{dR(t)}{dt} = 2.0467(\phi - \frac{R(t)}{K}) + 0.099)W_t$$

The following values (as shown in Table 3) were used for the parameters of the model:

Table 3: Parameters used in the algorithm

Parameter	Representation	Value
Initial value	X_0	171.20
Mean	μ	2.04
Variance	σ	0.99
Carrying capacity	K	900

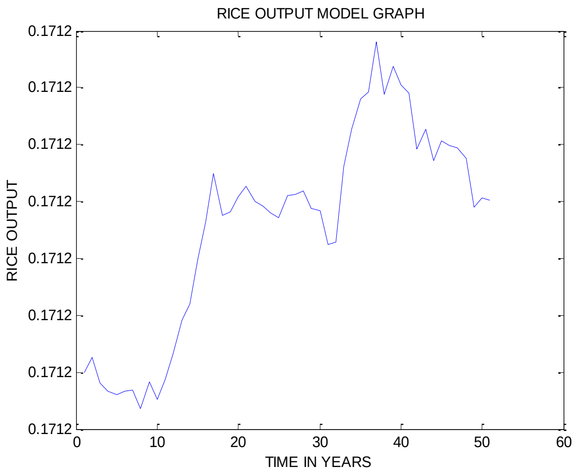

Fig. 1: Graph showing rice output

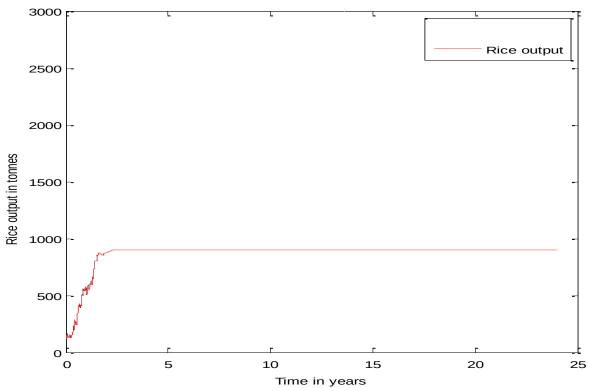

Fig. 2: Graph showing rice output with carrying capacity

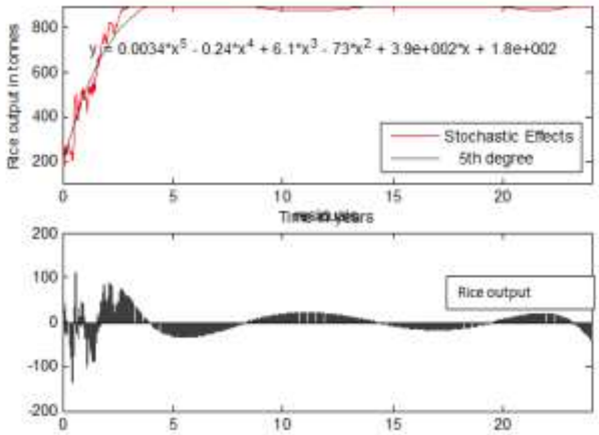

Fig. 3: Interpolating polynomial of simulated rice output with its residual

The basic variables that characterize SDEs are the drift and the volatility coefficients. We determined the drift and the volatility coefficients of the resulting stochastic model in equation (6) using SPSS. The output is shown in Table 2. The value of the mean of the mean output and the variance of the mean output are used as it is more appropriate for the model.
In Fig. 1 our plot shows the trend of Rice productions over the period of study which when view closely relate to the real data. The trend shows us the variance of the output as with regards to the productions. From the graph it shows a sharp rise in production at about the ten year follows with a slight downward output.
In Fig. 2, we observed that the rice output attained the carry capacity about the third year. This means that there has not been significant growth in rice production for the remaining of the period under study.
In order to predict future rice output using the model, we fitted a fifth degree interpolating polynomial y = 0.0034x^5 – 0.24x^4 + 6.1x^3 – 7.3x^2 + 39002x + 18002 into the curve of our graph. We observed from the graph that forecasting rice output with this approximation to the model will perform better with increase in years as the intensity of the variation reduces over time.
In order to validate our model a student T test is done to comparing our actual rice output data and the simulated rice data and our result shows no significance difference.

Conclusion and Recommendations

In this work, we formulate a stochastic differential equation model on the variability of rice production in Benue state. This was possible by carrying out analysis on data for rice production in the state gotten from BNARDA to input as our model parameters.
The model formulated is nonlinear with implicit solution; hence we used Matlab software in solving our model using the Stochastic Runge-Kutta scheme. Result from our model shows us the curve movement of the rice production in the state and the effect of the carry capacity, also predictions of rice production process from our model were achieved by using the Matlab software to fit a fifth order polynomial into the curve.

Clearly from our result we could see the trend in rice production as it relate to reality and also relate to the data collected in which the output is unstable. The result also shows that one of the major factors for higher rice production depends on the carrying capacity. Comparing our result also using the student T test also shows us clearly that our model follows the trend.

Result of our model agrees with the work of Henri (2007) and Rajoti (2014). However in our work we used parameters from the study area rather than the assumed values used in other work.

Following the study carried out in this work, we recommend that future researchers should focus on how to capture the dynamics of rice production in the state with full considerations of the factors affecting rice production. This is because a major challenge encounter at the course work is the fact that there are no documented data about some relevant factors that could contribute to variation in rice production such as pest, natural deserters and others. Consequently we recommend that proper data keeping by relevant bodies to enable good research should be encouraged.

The model proposed for the variability of Rice production in Benue State is recommended to be put in use in the state since it will give the state a better knowledge about rice production and possible cause for high or low rice production variability in the state which could be used for strategic planning.

Also, we recommend that further research may assume that farm inputs are not constant by factoring in the effects of changes in farm inputs in rice production in Benue State.

Conflict of Interest

Authors declare that there is no conflict of interest.

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APPENDIXES

T-Test Table

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	ACTUAL RICE OUTPUT	.252517	23	.0431777	.0090032
	MODEL RICE OUTPUT	.171200	23	.0000000	.0000000

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	ACTUAL RICE OUTPUT & MODEL RICE OUTPUT	23	.	.

Paired Samples Test

	Paired Differences			
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower
Pair 1 ACTUAL RICE OUTPUT - MODEL RICE OUTPUT	.0813174	.0431777	.0090032	.0626460

Paired Samples Test

	Paired Differences			
	95% Confidence Interval of the Difference Upper	t	Df	Sig. (2-tailed)
Pair 1 ACTUAL RICE OUTPUT - MODEL RICE OUTPUT	.0999888	9.032	22	.000